STABILITY OF A CYLINDRICAL SHELL UNDER PROGRAMMED VARIATION OF AN AXIAL COMPRESSIVE FORCE UNDER CREEP CONDITIONS

L. M. Kurshin and V. T. Shcherbakov

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The stability of cylindrical shells with initial deflection is examined under creep with a programmed load. Data are presented from an experimental study of the stability of shells under creep which are compared to the results of a calculation.

Most shells used in construction of aircraft operate under creep conditions with variable loads and heat temperatures. In this work the stability of a cylindrical shell with initial deflections and variable axial load at constant temperature is calculated. The simplest programs for varying compressive forces are treated (Fig. 1). Results are presented of an experimental study of the stability of cylindrical shells under creep with a programmed load.

A derivation of the disturbed-motion equations for a shell under creep conditions with initial deflection was given in [1] for a constant intensity of ground-state stresses. Linearized physical equations were used, and geometric nonlinearity was taken into account. The flow equations for a shell will be written in this work in the same form as in [1], but with a variable intensity of the ground-state stresses.

Suppose the creep equation has the form

$$\dot{p}_i = g(p_i, \sigma_i)\,\sigma_i \tag{1}$$

where $\dot{p}_i$ and σ_i are the creep and stress flow rates, and let a flow-theory-type equation

$$\dot{p}_{ij} = (^3/_2)\, g(p_i, \sigma_i)\, \sigma_{ij}^{**}, \quad p_{ij} = \varepsilon_{ij} - (^1/_2\, G^{-1})\, \sigma_{ij}^{**} \tag{2}$$

hold between the components of the creep flow rate tensor $\dot{p}_{ij}$ and stress deviator σ_{ij}^{**}.

We assume that the stressed state of the shell consists of a momentless ground and some disturbed state

$$\sigma_{ij}(t) = \sigma_{ij} + \delta\sigma_{ij}(t), \quad \dot{p}_{ij}(t) = \dot{p}_{ij} + \delta\dot{p}_{ij}(t) \tag{3}$$

Redistribution of stresses and displacements occurs during the creep process. We assume that additions due to disturbances are small and that the linearized equations [2]

$$\begin{gathered} \delta\dot{p}_i = \sigma_i \frac{\partial g}{\partial p_i}\delta p_i + \sigma_i \frac{\partial g}{\partial \sigma_i}\delta\sigma_i + g\delta\sigma_i \\ \delta\dot{\varepsilon}_{ij} - \frac{1}{2G}\delta\dot{\sigma}_{ij}^{**} = \frac{3}{2} g\delta\sigma_{ij}^{**} + \frac{3}{2}\sigma_{ij}^{**}\left(\frac{\partial g}{\partial \sigma_i}\delta\sigma_i + \frac{\partial g}{\partial p_i}\delta p_i\right) \end{gathered} \tag{4}$$

are valid for magnitudes characterizing the deviation from the stressed state.

Integrating Eq. (4) with respect to the variable σ_i and writing the strain of a tapered shell related to the deviation from the ground state, the moments, and additional forces in the middle surface, and carrying out the transformations as in [1], we obtain a system of equations describing the behavior of shells under creep:

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$$
\begin{gathered}
\Delta\Delta\Phi + e^{-\varkappa}\int_0^{\varkappa^*} e^{\varkappa} S(\Lambda_1\Lambda_1\Phi)\,d\varkappa - B\left[\Gamma(w, w_0) - e^{-\varkappa}\int_0^{\varkappa^*} e^{\varkappa}\Gamma(w, w_0)\,d\varkappa\right] = 0 \\
U(w, w_0, \Phi) + e^{-\varkappa}\int_0^{\varkappa^*} e^{\varkappa} D\Delta\Delta(w - w_0)\,d\varkappa - e^{-\varkappa}\int_0^{\varkappa^*} e^{\varkappa} S\left(-U(w, w_0, \Phi) - \right. \\
\left. - \frac{3}{4} D\Lambda\Lambda(w - w_0) - e^{-\varkappa}\int_0^{\varkappa^*} e^{\varkappa} D\left(\Delta\Delta - \frac{3}{4}\Lambda\Lambda\right)(w - w_0)\,d\varkappa\right) d\varkappa = 0
\end{gathered} \tag{5}
$$

The stress, deflection, and initial deflection functions are denoted in Eqs. (5) by Φ, w, and w_0, while D denotes the cylindrical rigidity of the shell. The operators Δ, Λ, Λ_1, U, and Γ have the form

$$
\begin{gathered}
\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad \Lambda = \alpha_{11}\frac{\partial^2}{\partial x^2} + 2\alpha_{12}\frac{\partial^2}{\partial x\,\partial y} + \alpha_{22}\frac{\partial^2}{\partial y^2} \\
\Lambda_1 = \alpha_{11}\left(\frac{\partial^2}{\partial y^2} - \frac{1}{2}\frac{\partial^2}{\partial x^2}\right) + \alpha_{22}\left(\frac{\partial^2}{\partial x^2} - \frac{1}{2}\frac{\partial^2}{\partial y^2}\right) - 3\alpha_{12}\frac{\partial^2}{\partial x\,\partial y} \\
U(w, w_0, \Phi) = -D\Delta\Delta(w - w_0) - \frac{1}{R_1}(N_{11}{}^0 + \Phi_{yy}) - \\
- \frac{1}{R_2}(N_{22}{}^0 + \Phi_{xx}) + \Phi_{yy}w_{xx} + \Phi_{xx}w_{yy} - 2\Phi_{xy}w_{xy} + 2h\sigma_i\Lambda w + q \\
\Gamma(w, w_0) = \left(\frac{1}{R_1}\frac{\partial^2}{\partial y^2} + \frac{1}{R_2}\frac{\partial^2}{\partial x^2}\right)(w - w_0) + w_{xy}{}^2 - w_{xy}{}^{02} - w_{xx}w_{yy} + w_{xx}{}^0 w_{yy}{}^0
\end{gathered}
$$

Equations (5) coincide with equations from [1], though the dimensionless parameter due to creep flow has the form

$$
\varkappa = \int_0^{\varkappa} (E/\sigma_i)\,dp_i
$$

In the particular case when σ_i = const, we have $\varkappa = (E/\sigma_i)p_i = \xi$.

In the case of axial compression of a cylindrical shell 2h thick,

$$
\begin{gathered}
\sigma_{11} = -\sigma,\ \sigma_{12} = \sigma_{22} = 0,\ \sigma_i = \sigma,\ \alpha_{11} = -1,\ \alpha_{12} = \alpha_{22} = 0 \\
R_1 = \infty,\ R_2 = R,\ N_{11} = -2\sigma h,\ N_{12} = N_{22} = 0
\end{gathered} \tag{6}
$$

For the creep law of Eq. (1) with

$$
g = A\sigma_i^{n-1} \tag{7}
$$

the values of a, b, and $S(\varphi)$ are given by

$$
a = 0,\ b = n - 1,\ S(\varphi) = (n - 1)\varphi \tag{8}
$$

Equations (5), as a result of Eqs. (6)-(8), take the form

$$
\begin{gathered}
\Delta\Delta\Phi + e^{-\varkappa}\int_0^{\varkappa^*} e^{\varkappa}(n - 1)\Lambda_1\Lambda_1\Phi\,d\varkappa - B\left[\Gamma(w, w_0) - e^{-\varkappa}\int_0^{\varkappa^*} e^{\varkappa}\Gamma(w, w_0)\,d\varkappa\right] = 0 \\
U(w, w_0, \Phi) = e^{-\varkappa}\int_0^{\varkappa^*} e^{\varkappa} D\left(\Delta\Delta - \frac{3}{4}\Lambda\Lambda\right)(w - w_0)\,d\varkappa - \frac{3n}{4} De^{-n\varkappa}\int_0^{\varkappa^*} e^{n\varkappa}\Lambda\Lambda(w - w_0)\,d\varkappa
\end{gathered} \tag{9}
$$

The elastic state of a shell with initial deflection w_0 is described by the nonlinear equations

$$
\begin{gathered}
-D\Delta\Delta(w - w_0) - \frac{1}{R}\Phi_{xx} + \Phi_{yy}w_{xx} + \Phi_{xx}w_{yy} - 2\Phi_{xy}w_{xy} = 0 \\
\frac{1}{B}\Delta\Delta\Phi = \frac{1}{R}(w - w_0)_{xx} + w_{xy}{}^2 - w_{xx}w_{yy} - [w_{xy}{}^{02} - w_{xx}{}^0 w_{yy}{}^0] \\
B = 2Eh
\end{gathered} \tag{10}
$$

An approximate solution of Eqs. (10) if the initial deflection is given in the form

$$
w_0 = f_1{}^0 \sin\frac{\alpha x}{2}\sin\frac{my}{R} + f_2{}^0\cos\alpha x \tag{11}
$$

and if the solutions are found in the form

$$w = f_1 \sin\frac{\alpha x}{2} \sin\frac{my}{R} + f_2 \cos\alpha x + f_3 \tag{12}$$

leads to the equations [3] for the deflection amplitudes $\zeta_1 = f_1/2h$ and $\zeta_2 = f_2/2h$, which are the initial conditions for solving the creep problem for shells with initial deflections ζ_1^{0}, ζ_2^{0}.

We find the solutions of Eqs. (9) in the form

$$\begin{aligned} w &= \varphi_1(\varkappa) f_1 \sin\frac{\alpha x}{2} \sin\frac{my}{R} + \varphi_2(\varkappa) f_2 \cos\alpha x + \varphi_3(\varkappa) f_3 \\ \Phi &= \Psi_1(\varkappa) C_1 \cos\alpha x + \Psi_2(\varkappa) C_2 \cos\frac{2my}{R} \\ &+ \Psi_3(\varkappa) C_3 \sin\frac{\alpha x}{2} \sin\frac{my}{R} + \Psi_4(\varkappa) C_4 \sin\frac{3\alpha x}{2} \sin\frac{my}{R} \end{aligned} \tag{13}$$

Integrating with respect to the coordinates x and y in the sense of Bubnov—Galerkin, we obtain a system of nonlinear integral equations related to the variable $\varkappa$,

$$a_1\varphi_1 + a_2\varphi_2 + a_3 = 0, \quad b_2\varphi_1 + b_1\varphi_2 + b_3 = 0 \tag{14}$$

Solving the equations for the linear parts of the deflections under creep φ_1 and φ_2, we obtain the system

$$\begin{aligned} &\varphi_1 = \frac{a_2b_3 - a_3b_1}{a_1b_1 - a_2b_2}, \qquad \varphi_2 = \frac{a_3b_2 - a_1b_3}{a_1b_1 - a_2b_2} \\ &a_1 = \zeta_1 [g_1 - (4/3)\, p - (\eta / 4)\, g_2\zeta_1^{02} + (1 / \upsilon^2)\, \zeta_2^{0}] \\ &a_2 = [(4\zeta_1^{0}\zeta_2) / (\upsilon^2)]\, g_3 \\ &a_3 = -\zeta_1^{0} [g_1 - (4 / \upsilon^2\lambda_1^2)\, \zeta_2^{0}] - (1 / \upsilon^2)\, g_4\varphi_1\varphi_2\zeta_1\zeta_2 + (\eta / 4)\times \\ &\times g_2\varphi_1^3\zeta_1^3 + (16\eta / \upsilon^2)\, g_9\varphi_1\varphi_2^2\zeta_1\zeta_2^2 - (k_2 / 4)\, g_8\varphi_1\zeta_1 J_{22} + (2k_3 / \\ &/\upsilon^2\lambda_1^2)\, [2\varphi_2\zeta_2 - (1 / 2\eta)]\, g_5 + (k_4 / \upsilon^2)\, g_6\varphi_1\zeta_1 - [(16k_5\eta) / \\ &/(81\, \upsilon^2\lambda_2^2)]\, \varphi_2\zeta_2 J_{54} - (g_8 / 3)\, [(4/3)\, \lambda_1^2 - 1]\, J_{13} - (n / 3)\, g_8 J_{23} \\ &b_1 = \zeta_2 [g_7 - (16/3)\, p], \; b_2 = (2\zeta_1^{0}\zeta_1 g_3) / \upsilon^2 \\ &b_3 = -g_7 / \zeta_2^{0} + (1 / 4\upsilon^2)\, \zeta_1^{02} - (g_4\varphi_1^2\zeta_1^2) / (4\upsilon^2) + (8\eta g_9\times \\ &\times\varphi_1^2\varphi_2\zeta_1^2\zeta_2)\, \upsilon^2 - (16/9)\, g_8\, (J_{11} + 3nJ_{21}) + (2k_3 / \upsilon^2\lambda_1^2)\times \\ &\times(J_{33} - 4\eta J_{34})\, \varphi_1\zeta_1 - (k_4 / g_8)\, g_6 - (8\eta k_5 / 81\upsilon^2\lambda_2^2)\, \varphi_1\zeta_1 J_{54} \\ &g_1 = (4/9)\, \upsilon^2\eta\lambda_1^2 + 1 / (\upsilon^2\eta\lambda_1^2), \; g_2 = \upsilon^2 + 1 / \upsilon^2, \; g_3 = 1 / \\ &/\lambda_1^2 - 4\eta\zeta_2^{0}X, \; g_4 = 1 + 8 / \lambda_1^2, \; g_5 = J_{33} - 4\eta J_{34}, \; g_6 = J_{41} - \\ &-(\eta / 4)\, J_{42}, \quad g_7 = (64 / 9)\, \upsilon^2\eta + 1 / (\upsilon^2\eta), \quad g_8 = \upsilon^2\eta, \quad g_9 = 1 / \\ &/\lambda_1^2 + 1 / (81\lambda_2^2) \quad \lambda_1 = (1 + \upsilon^2) / \upsilon^2, \; \lambda_2 = (1 + 9\upsilon^2) / (9\upsilon^2) \end{aligned} \tag{15}$$

We introduce in the expressions for a_i and b_i the notation

$$\begin{aligned} &J_{ij} = e^{-k_i\varkappa} \int_0^{\varkappa^*} e^{k_i\varkappa} H_j(\varkappa)\, d\varkappa \\ &H_1 = \varphi_2\zeta_2 - \zeta_2^{0}, \; H_2 = \varphi_1^2\zeta_1^2 - \zeta_1^{02} \\ &H_3 = \varphi_1\zeta_1 - \zeta_1^{0}, \; H_4 = \varphi_1\varphi_2\zeta_1\zeta_2 - \zeta_1^{0}\zeta_2^{0} \\ &k_1 = 1, \; k_2 = n, \; k_3 = 1 + (n - 1)\, (\upsilon^2 / 2 - 1)^2 / (\upsilon^2 + 1)^2 \\ &k_4 = (n + 3) / 4, \; k_5 = 1 + (n - 1)\, (9\upsilon^2 / 2 - 1)^2 / (9\upsilon^2 + 1)^2 \\ &\upsilon = (\alpha R) / (2m), \; \eta = (3/8)\, \beta^{*2}, \; p = (3R\sigma) / (4Eh) \end{aligned}$$

We differentiate the system (14) with respect to p,

$$\frac{\partial a_1}{\partial p}\varphi_1 + \frac{\partial a_2}{\partial p}\varphi_2 + \frac{\partial a_3}{\partial p} = 0, \qquad \frac{\partial b_2}{\partial p}\varphi_1 + \frac{\partial b_1}{\partial p}\varphi_2 + \frac{\partial b_3}{\partial p} = 0 \tag{16}$$

We compile the determinant of the system,

$$\begin{aligned} &M = [(a_1 / \zeta_1 + T_2)\, (b_1 / \zeta_2 + T_3) - 2\, (a_2 / \zeta_2 + T_1)^2]\, \varphi_1\varphi_2 \\ &T_1 = -(1 / 2\upsilon^2) g_4\varphi_1\zeta_1 + (16 / \upsilon^2)\eta\varphi_1\varphi_2\zeta_1\zeta_2 X + (2 / \lambda_1^2) \\ &\times g_5k_3 - (8\eta k_5 / 81\upsilon^2\lambda_2^2)\, J_{54} \\ &T_2 = -(1 / \upsilon^2)\, g_4\varphi_2\zeta_2 + (3/4)\eta g_2\varphi_1^2\zeta_1^2 + (16 / \upsilon^2)\eta\varphi_2^2\zeta_2^2 g_9 - (k_2 / 4)\, g_8 J_{22} + (k_4 / \upsilon^2)\, g_6 \\ &T_3 = (8 / \upsilon^2)\eta g_9\varphi_1^2\zeta_1^2 \end{aligned} \tag{17}$$

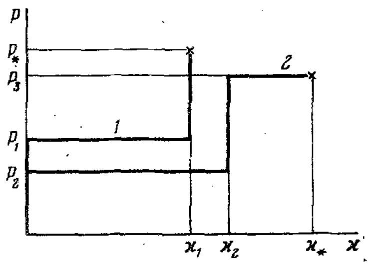

Fig. 1

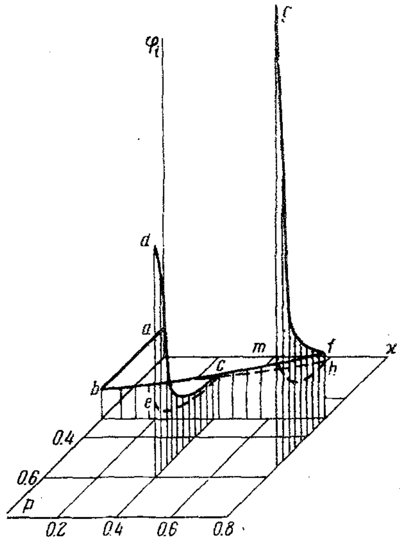

Fig. 2

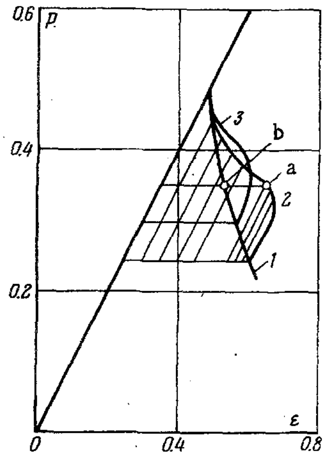

Fig. 3

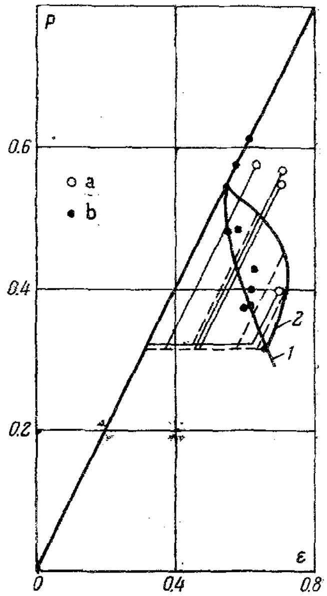

Fig. 4

The critical time $\varkappa$ is found either by setting M equal to zero or by imposing a minimum condition on it.

A calculation was performed for a shell under creep using Eqs. (15) for two cases of varying the axial compressive load. Figure 2 depicts the dependence of deflections under creep calculated for initial deflection values of $\zeta_0 = 3\zeta_2^{\circ} = 0.2$, and $\zeta_{0_k} = 3\zeta_1^{\circ} = 0.05$.

In the three-dimensional coordinate system φ_i, $\varkappa$, p (Fig. 2) the curves abcd and abcfg correspond to antisymmetric deflections φ_1 and abce and abchm, to symmetric deflections φ_2 under creep, respectively, for the two values of the parameter $\varkappa_1$ characterizing the shell delay time under constant load ($\varkappa_1 = 0.4$, $\varkappa_1 = 0.8$). The straight line ab corresponds to an elastic load of a shell with compressive stresses p = 0.3. The curves bc, bcf and bc, bch correspond to a growth in the antisymmetric φ_1 and symmetric φ_2 deflections for a constant compressive load $p = 0.3$ and the curves cd, fg, and ce, hm, to a growth in φ_1 and φ_2 under instantaneous loading of a shell with compressive stresses until stability is lost. The results of a calculation of the critical values of load p_* for different values of the parameter $\varkappa_1$ are depicted for program I (Fig. 1) in Fig. 3.

Calculations using Eqs. (15) and (17) were carried out on a computer for initial deflection values $\zeta_0 = 0.2$ and $\zeta_{0k} = 0.05$. Curve 2 corresponds to the case $p_1 = 0.224$ and curve 3, to the case $p_1 = 0.3$. The size of p_1 is due to the level of constant stresses at which creep strain of shells accumulate (Fig. 1). The total critical strain values ε are determined from the equation

$$\varepsilon = p_* + \varkappa_1 p_1$$

We may note in considering the results of the calculation of the critical strain of shells under creep conditions, depicted in Fig. 3 by curves 2 and 3, that a distinct additional instantaneous loading is required in order for a shell to collapse as a function of the magnitude of the preliminary creep strain. The magnitude of this loading is noticeably decreased only for significant accumulated creep strain.

We may also note that curve 2 (or 3) corresponding to critical parameters p and ε for the case of a program with loading (program I, Fig. 1) lies above curve 1, which corresponds to the critical parameters under creep with constant compressive forces. Thus, the critical strain is greater for a collapse with a loading (point a) than in a collapse under creep conditions with constant load equal in magnitude to the critical compressive load which shells collapsing under a load experience (point b).

Calculations of critical strains were also carried out for load programs of the type of program 2 (Fig. 1). Results are given below of a calculation of the critical values $\varkappa_*$ for given compressive loads $p_2 = 0.244$ and $p_3 = 0.375$ and different fixed $\varkappa_2$:

$\varkappa_2$	0.20	0.30	0.37	0.40	0.60	0.70	0.75	0.78
$\varkappa_*$	0.47	0.53	0.56	0.59	0.70	0.76	0.80	0.82

It is of interest to compare the results of the current work with data from an experimental study of shell stability under creep conditions with programmed load. The calculation requires that we specify the values of the initial shell deflections. We used a technique proposed in [3, 4] for this purpose.

A total of four shells machined from D16T material at T = 250°C (radius R = 88 mm, thickness 2h = 0.5 mm, and length l = 425 mm) were tested under creep conditions with programmed load. The time, load, and approach of the end faces of the shell were measured in the experiment (to determine the axial creep strain). A variation program for an axial load of type I was realized.

After being heated to a given temperature the shell was loaded to a value p of 0.32 at which it was held until a given magnitude of creep strain accumulated and was then rapidly loaded with an axial force until it collapsed. The shell lost stability "in crashes" with the formation of bands of rhombic depressions along the periphery. The test results are depicted in Fig. 4 by points a, which represent the dependence of total critical strain ε on axial load p. Test data for the same shells under constant levels of compressive loads are depicted in Fig. 4 by points b under creep conditions.

The initial deflections $\zeta_0 = 0$ and $\zeta_{0k} = 0.68$, which were selected from data of an elastic experiment according to the technique described in [4], were introduced in calculating the critical strains. The critical time and total critical strain under creep for shells that have undergone tests were determined using Eqs. (15) and (17). The results of the calculations are depicted in Fig. 4 by curves 1 and 2. Curve 1 corresponds to critical strain under creep conditions for a constant axial load and curve 2, to critical strains of shells under creep that have lost stability with loading.

The good agreement between the calculated curves and experimental data allows us to conclude that it is sufficient to have results from elastic tests in order to calculate the stability of shells under creep conditions with an axial load varying according to some program. These data allow us to select the initial deflections that can then be introduced into a calculation of the stability of a shell under creep with an axial load varying according to a program.

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